WEST BENGAL STATE UNIVERSITY

B.Sc. Honours/Programme 1st Semester Examination, 2019

MTMHGEC01T/MTMGCOR01T-MATHEMATICS (GE1/DSC1)

DIFFERENTIAL CALCULUS

Time Allotted: 2 Hours

Full Marks: 50

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1. Answer any ***five*** questions from the following: $2\times 5 = 10$

(a) Evaluate the right hand and left hand limits of the function $f(x)=\frac{|x|}{x}$ at the point $x=0$. Examine whether the function has a limit at 0.

(b) Find the points of discontinuity of the function $f(x)=\dfrac{x-1}{(x^2-1)(x-1)x}$.

(c) Verify Rolle's theorem for the function $f(x)=x^2-5x+10$ on [2, 3].

(d) Investigate the extremum for the function $f(x)=2x^3-15x^2+42x+10$.

(e) Show that $\dfrac{\sin\alpha-\sin\beta}{\cos\alpha-\cos\beta}=\cot\theta$, where $0<\alpha<\theta<\beta<\frac{\pi}{2}$.

(f) Show that the function $f(x)=\sqrt{\dfrac{x^{1/3}+y^{1/3}}{x^{1/4}+y^{1/4}}}$ is homogeneous and find its degree.

(g) Find the points on the curve $y=x^2+3x+4$, where the tangents pass through the origin.

(h) If $y=\sin(m\sin^{-1}x)$, show that $(1-x^2)y_2-xy_1+m^2y=0$.

2. (a) Show that the limit that $\lim\limits_{x\to 0}\sin\frac{1}{x}$ does not exist. 2

(b) If two functions f and g are continuous at a point c, then show that $f+g$ is also continuous at c. 3

(c) Discuss the continuity of the function $f(x)=|x-3|$ at $x=3$ and find $f'(3)$, if exists. 2+1

3. (a) If a function f is differentiable at some point c in its domain, then prove that it is also continuous at c. Give a suitable example to show that the converse of the above result is not true. 3

(b) If $u = \tan^{-1}\dfrac{x^3 + y^3}{x - y}$, then prove that $x\dfrac{\delta u}{\delta x} + y\dfrac{\delta u}{\delta y} = \sin 2u$. 3

(c) Find the equation of the normal to the curve $x^2 - y^2 = a^2$ at the point $(a\sqrt{2}, a)$. 2

4. (a) If $y^{1/m} + y^{-1/m} = 2x$, prove that $(x^2 - 1)y_{n+2} + (2n+1)xy_{n+1} + (n^2 - m^2)y_n = 0$. 4

(b) State and prove Euler's theorem on homogeneous functions. 4

5. (a) Find the radius of curvature at $(\frac{1}{4}, \frac{1}{4})$ of the curve $\sqrt{x} + \sqrt{y} = 1$. 3

(b) State and prove Lagrange's mean value theorem. Write the geometrical interpretation of this theorem. 4+1

6. (a) Find the Taylor's series expansion of the function $f(x) = \sin x,\ x \in \mathbb{R}$. 5

(b) Determine the asymptotes of the curve $x = \dfrac{2t}{t^2 - 1},\ y = \dfrac{(1+t)^2}{t^2}$. 3

7. (a) Verify Rolle's theorem for the function $f(x) = x(x+3)e^{-x/2}$ in $[-3, 0]$. 4

(b) If $f: \mathbb{R} \longrightarrow \mathbb{R}$ is a differentiable function such that $f'(x) = 0$ everywhere then show that $f(x)$ is a constant function on $\mathbb{R}$. 4

8. (a) If $f(h) = f(0) + hf'(0) + \dfrac{h^2}{2!}f''(\theta h),\ 0 < \theta < 1$, find θ when $h = 1$ and $f(x) = (1 - x)^{3/2}$. 3

(b) Discuss maxima and minima of the function $f(x) = (\frac{1}{x})^x,\ x > 0$, if there be any. 5

9. (a) If $u = f(x, y),\ x = r\cos\theta,\ y = r\sin\theta$, then prove that 6

$$\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 = \left(\frac{\partial u}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial u}{\partial \theta}\right)^2$$

(b) If $f(x, y) = \dfrac{xy(x^2 - y^2)}{x^2 + y^2}$ for $(x, y) \neq (0, 0)$ and $f(0, 0) = 0$ then find $f_x(0, 0)$ and $f_y(0, 0)$. 1+1

———×———