लक्ष्यं विश्वमानम्

WEST BENGAL STATE UNIVERSITY

B.Sc. Honours/Programme 1st Semester Examination, 2020, held in 2021

MTMHGEC01T/MTMGCOR01T-MATHEMATICS (GE1/DSC1)

DIFFERENTIAL CALCULUS

Time Allotted: 2 Hours | Full Marks: 50

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1. Answer any ***five*** questions from the following: $2\times5 = 10$

(a) Evaluate $\lim\limits_{x\to 0}(1+3x)^{\frac{2}{x}}$

(b) Find $\lim\limits_{x\to 2}\sqrt{x-2}$ if it exists.

(c) Show that $f(x)=2x^2+3x+5$ is continuous for any real number x.

(d) Find $\frac{dy}{dx}$ if $(\cos x)^y=(\sin y)^x$.

(e) If $y=\frac{x}{x+1}$, show that $y_5(0)=5!$.

(f) At what point is the tangent to the parabola $y=x^2$ parallel to the straight line $y=4x-5$.

(g) Find the points of extremum value of the function $f(x)=\sin x(1+\cos x)$ in $[0, 2\pi]$.

(h) If $f(x, y)=x\log y$ then show that $f_{xy}=f_{yx}$.

(i) Find the asymptotes of the curve $x^3-6x^2y+11xy^2-6y^3+x+y+5=0$.

(j) Find the radius of curvature of the curve $xy=12$ at (3, 4).

2. (a) A function f is defined as follows: 4

$$f(x)=\begin{cases} x^2+ax & , \text{ if } 0\le x<1 \\ 3-bx^2 & , \text{ if } 1\le x\le 2 \end{cases}$$

If $\lim\limits_{x\to 1} f(x)=4$, find the value of a and b.

(b) If $f, g : D \to \mathbb{R}$ are two functions such that $\lim_{x\to c} f(x)$ and $\lim_{x\to c} g(x)$ exists finitely, then prove that $\lim_{x\to c} [f(x)+g(x)] = \lim_{x\to c} f(x).\ \lim_{x\to c} g(x)$.

3. (a) State and prove Lagrange's mean value theorem. 5

(b) If $f(x, y) = \tan^{-1}\dfrac{y}{x} + \sin^{-1}\dfrac{y}{x}$, find the value of $x\dfrac{\partial f}{\partial x} + y\dfrac{\partial f}{\partial y}$ at the point (1, 1). 3

4. (a) If $y = e^{a\sin^{-1}x}$, prove that $(1-x^2)y_{n+2} - (2n+1)x\,y_{n+1} + (m^2-n^2)y_n = 0$. 5

(b) Find the radius of curvature of the cycloid $x = a(\theta - \sin\theta)$, $y = a(1-\cos\theta)$ at any point θ. 3

5. (a) Show that the function f is continuous at $x=1$ but not differentiable at $x=1$ where 4

$$f(x) = \begin{cases} x+1 , & \text{if } 0 \le x < 1 \\ 3-x , & \text{if } 1 \le x \le 2 \end{cases}$$

(b) Find the points on the curve $y = 2x^3 - 15x^2 + 34x - 20$ where the tangents are parallel to the straight line $y + 2x = 0$. 4

6. (a) If $f(x, y) = \begin{cases} \dfrac{x^3 y}{x^2+y^2} , & x^2+y^2 \neq 0 \\ 0 , & x=0,\ y=0 \end{cases}$ 5

prove that $f_{xy}(0, 0) \neq f_{yx}(0, 0)$.

(b) Find the nature of double points of the curve $(2y+x+1)^2 = 4(1-x)^5$. 3

7. (a) Determine the points of discontinuities of the function 4

$$f(x) = \begin{cases} \sin\frac{1}{x} , & x \le 0 \\ 2x , & 0 < x < 1 \\ 0 , & x = 1 \\ \frac{x^2-1}{x-1} , & 1 < x \end{cases}$$

(b) Prove that $\dfrac{x}{1+x} < \log(1+x) < x$ for all $x > 0$. 4

8. (a) Determine the Taylor's series expansion of $f(x) = \cos x$. 5

(b) If a function f is differentiable on [0, 1] show that the equation $f(1) - f(0) = \dfrac{f'(x)}{2x}$ has at least one root in (0, 1). 3

9. (a) If $u = f(x, y)$ and $x = r\cos\theta$, $y = r\sin\theta$ then prove that

$$\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 = \left(\frac{\partial u}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial u}{\partial \theta}\right)^2$$

(b) Show that the area of a rectangle inscribed in a circle is the maximum when it is a square. 4

10.(a) A function f is thrice differentiable on $[a, b]$ and $f(a) = 0 = f(b)$ and $f'(a) = 0 = f'(b)$. Prove that there is a number c in $[a, b]$ such that $f'''(c) = 0$. 3

(b) If $u = f(y - z,\ z - x,\ x - y)$ then show that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$. 5

11.(a) Show that the radius of curvature at any point (r, θ) on the curve $r = a(1 - \cos\theta)$ varies as $\sqrt{r}$. 5

(b) If $f(x) = 2|x| + |x - 2|$, find $f'(1)$. 3

12.(a) Find the asymptotes of the following curve: 5

$$x = \frac{t^2}{1 + t^3} \quad , \quad y = \frac{t^2 + 2}{1 + t}$$

(b) If $\lim_{x \to 0} \frac{\sin 2x + a\sin x}{x^3}$ be finite, find the value of 'a' and the limit. 3

13.(a) State and prove Leibnitz's theorem on successive differentiation. 5

(b) Find the radius of curvature of the curve $y = xe^{-x}$ at its maximum point. 3

N.B. : *Students have to complete submission of their Answer Scripts through E-mail / Whatsapp to their own respective colleges on the same day / date of examination within 1 hour after end of exam. University / College authorities will not be held responsible for wrong submission (at in proper address). Students are strongly advised not to submit multiple copies of the same answer script.*

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