WEST BENGAL STATE UNIVERSITY

B.Sc. Honours/Programme 1st Semester Examination, 2022-23

MTMHGEC01T/MTMGCOR01T-MATHEMATICS (GE1/DSC1)

DIFFERENTIAL CALCULUS

Time Allotted: 2 Hours — Full Marks: 50

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.
All symbols are of usual significance.

Answer Question Number 1 and any *five* from the rest

1. Answer any ***five*** questions from the following: $2\times5=10$

(a) Examine whether the limit $\lim\limits_{x\to3}\frac{[x]}{x}$ exists, where $[x]$ represents the greatest integer less or equal to x.

(b) $f(x)=\begin{cases} x+1 & \text{when } x\leq1 \\ 3-ax & \text{when } x>1 \end{cases}$

For what value of a, will f be continuous at $x=1$.

(c) For the function $f(x)=|x|;\ x\in\mathbb{R}$ show that $f'(0)$ does not exists.

(d) Show that the function $f(x)=4x^2-6x-11$ is increasing at $x=4$.

(e) Find the point on the curve $y=x^3-6x+7$ where the tangent is parallel to the straight line $y=6x+1$.

(f) Find the asymptotes of the curve $xy^2-yx^2-(x+y+1)=0$.

(g) Examine the continuity of the function at $(0, 0)$

$$f(x, y)=\begin{cases} \frac{xy}{x^2+y^2}, & (x, y)\neq(0, 0) \\ 0, & (x, y)=(0, 0) \end{cases}$$

(h) Show that the function $f(x, y)=\frac{x^{1/4}+y^{1/4}}{x^{1/5}+y^{1/5}}$ is homogeneous in x and y. Find its degree.

(i) If $u=x\log y$, then show that $u_{xy}=u_{yx}$.

2. (a) If f is an even function and $f'(0)$ exists, then show that $f'(0)=0$. 4

(b) Discuss the continuity of f at $x=1$ and $x=2$ where $f(x)=|x-1|+|x-2|$. 4

3. (a) If $x+y=e^{x-y}$, show that $\frac{d^2y}{dx^2}=\frac{4(x+y)}{(x+y+1)^3}$ 4

(b) State and prove Lagrange's Mean Value Theorem. 4

4. (a) Find the slope of the curve $x^{2/3}+y^{2/3}=a^{2/3}$ at the point (x_1, y_1) and hence obtain the equation of the tangent at that point. 4

   (b) Verify Rolle's theorem for the function $f(x)=x\sqrt{4-x^2}$ is $0\le x\le 2$. 4

5. (a) Expand $f(x)=\sin x$ as a series of infinite terms. 5

   (b) If $y=\frac{x}{x+1}$, show that $y_5(0)=51$. 3

6. (a) If $f(x)=\log\frac{\sqrt{a+bx}-\sqrt{a-bx}}{\sqrt{a+bx}+\sqrt{a-bx}}$, find for what values of x, $\frac{1}{f'(x)}=0$. 4

   (b) Prove that $\lim\limits_{h\to 0}\frac{f(a+h)-2f(a)+f(a-h)}{h^2}=f''(a)$, provided that $f''(x)$ is continuous. 4

7. (a) Find the maxima and minima, if any, of $\frac{x^4}{(x-1)(x-3)^3}$. 2+2

   (b) Determine the values of a, b, c so that $\frac{a\sin x-bx+cx^2+x^3}{2x^2\log(1+x)-2x^3+x^4}$ may tend to a finite limit as $x\to 0$, and determine this limit. 3+1

8. (a) If $lx+my=1$ is a normal to the parabola $y^2=4ax$, then show that $al^3+2alm^2=m^2$. 4

   (b) If the tangent at (x_1, y_1) to the curve $x^3+y^3=a^3$ meets the curve again in (x_2, y_2), show that $\frac{x_2}{x_1}+\frac{y_2}{y_1}=-1$. 4

9. (a) Prove that the asymptotes of the curve $x^2y^2=a^2(x^2+y^2)$ form a square of side $2a$. 4

   (b) Show that for an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, the radius of curvature at an extremity of the major axis is equal to the half of the latus rectum. 4

10. (a) If V is a function r alone, where $r^2=x^2+y^2+z^2$, show that 4

$$\frac{\partial^2 V}{\partial x^2}+\frac{\partial^2 V}{\partial y^2}+\frac{\partial^2 V}{\partial z^2}=\frac{d^2V}{dr^2}+\frac{2}{r}\frac{dV}{dr}.$$

   (b) If $y=f(x+ct)+\phi(x-ct)$, show that $\frac{\partial^2 y}{\partial t^2}=c^2\frac{\partial^2 y}{\partial x^2}$. 4

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