लक्ष्यं विश्वमानम्

WEST BENGAL STATE UNIVERSITY

B.Sc. Honours/Programme 2nd Semester Examination, 2022

MTMHGEC02T/MTMGCOR02T-MATHEMATICS (GE2/DSC2)

Time Allotted: 2 Hours | Full Marks: 50

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1. Answer any ***five*** questions from the following: 2×5 = 10

(a) Test whether the equation $(\sin 2x - \tan y)\,dx = x\sec^2 y\; dy$ is exact or not?

(b) Find an integrating factor of the differential equation $(2x^2 + y^2 + x)\,dx + xy\,dy = 0$.

(c) Find the differential equation of the family of parabolas $y^2 = 4ax$, where a is an arbitrary constant.

(d) Verify if the following pair of functions are independent

$$e^x \quad , \quad 5e^x$$

(e) Given that $y_1(x)$, $y_2(x)$ and $y_3(x)$ are solutions of $\{D^2 + p(x)\,D + q(x)\}y = 0$, where $D \equiv \dfrac{d}{dx}$. Show that these solutions are linearly independent.

(f) Verify the integrability of the following differential equation:

$$yz\,dx = zx\,dy + y^2 dz$$

(g) Determine the order, degree and linearity of the following P.D.E:

$$\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial^2 z}{\partial x \partial y} + \left(\frac{\partial z}{\partial y}\right)^2 = 0$$

(h) Eliminate the arbitrary functions ϕ and ψ from $z = \phi(x+iy) + \psi(x-iy)$, where $i^2 = -1$.

2. (a) Determine the constant A of the following differential equation such that the equation is exact and solve the resulting exact equation: 4

$$\left(\frac{Ay}{x^3} + \frac{y}{x^2}\right)dx + \left(\frac{1}{x^2} - \frac{1}{x}\right)dy = 0$$

(b) Reduce the equation $\sin y \dfrac{dy}{dx} = \cos x\,(2\cos y - \sin^2 x)$ to a linear equation and hence solve it. 4

3. (a) Using the transformation $u = x^2$ and $v = y^2$ to solve the equation 4

$$xyp^2 - (x^2 + y^2 - 1)\,p + xy = 0 \quad , \quad \text{where } p = \frac{dy}{dx}$$

(b) Solve: $\dfrac{dy}{dx}+\dfrac{ax+hy+g}{hx+by+f}=0$

4. (a) Solve by the method of variation of parameters:

$$\frac{d^2y}{dx^2}+a^2y=\cos ax$$

(b) Show that e^x and xe^x are linearly independent solutions of the differential equation $\dfrac{d^2y}{dx^2}-2\dfrac{dy}{dx}+y=0$. Write the general solution of this differential equation. Find the solution that satisfies the condition $y(0)=1,\ y'(0)=4$. Is it the unique solution? 1+1+1+1

5. (a) Solve: $\{(5+2x)^2D^2-6(5+2x)D+8\}y=8(5+2x)^2$, where $D\equiv\dfrac{d}{dx}$. 4

(b) Solve the following equations: 4

$$\frac{dx}{dt}+4x+3y=t \quad ; \quad \frac{dy}{dt}+2x+5y=e^t$$

6. (a) Verify that the following equation is integrable, find its primitive: 5

$$zy\,dx+(x^2y-zx)\,dy+(x^2z-xy)\,dz=0$$

(b) Solve: $(4x^2y-6)\,dx+x^3\,dy=0$ 3

7. (a) Eliminate the arbitrary function ϕ from the relation $z=e^{my}\phi(x-y)$. 3

(b) Solve the PDE by Lagrange's method: 5

$$px(x+y)-qy(x+y)+(x-y)(2x+2y+z)=0$$

8. (a) Find the particular solution of the differential equation 4

$$(y-z)\frac{\partial z}{\partial x}+(z-x)\frac{\partial z}{\partial y}=x-y$$

which passes through the curve $xy=4,\ z=0$.

(b) Determine the points $(x,\ y)$ at which the partial differential equation 4

$$(x^2-1)\frac{\partial^2 z}{\partial x^2}+2y\frac{\partial^2 z}{\partial y\partial x}-\frac{\partial^2 z}{\partial y^2}=0$$

is hyperbolic or parabolic or elliptic.

9. (a) Solve: $(x^2+y^2+z^2)\,dx-2xy\,dy-2xz\,dz=0$ 4

(b) Solve in particular cases: 4

$$\frac{d^2y}{dx^2}+y=\sin 2x \quad ; \quad \text{when } x=0,\ y=0 \text{ and } \frac{dy}{dx}=0$$

N.B. : *Students have to complete submission of their Answer Scripts through E-mail / Whatsapp to their own respective colleges on the same day / date of examination within* 1 *hour after end of exam. University / College authorities will not be held responsible for wrong submission (at in proper address). Students are strongly advised not to submit multiple copies of the same answer script.*

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