WEST BENGAL STATE UNIVERSITY

B.Sc. Programme 5th Semester Examination, 2022-23

MTMGDSE01T-MATHEMATICS (DSE1)

Time Allotted: 2 Hours

Full Marks: 50

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1. Answer any *five* questions from the following: $2\times5 = 10$

(a) Is the set of vectors (1, 0, 0), (0, 1, 0) and (0, 0, 1) are linearly dependent? Justify your answer. 1+1

(b) What is the geometric meaning of the given transformation

$$\begin{pmatrix} X \\ Y \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

(c) Is any straight line passing through (0, 0, 0) in $\mathbb{R}^3$ a sub space of $\mathbb{R}^3$? Give reason.

(d) Write down the matrix form of the system of equations

$$a_1x + b_1y + c_1z = d_1$$
$$a_2x + b_2y + c_2z = d_2$$
$$a_3x + b_3y + c_3z = d_3$$

(e) Is the vectors (1, 2) and (–1, 2) and linearly independent in $\mathbb{R}^3$? Justify.

(f) Find the inverse of the matrix $\begin{pmatrix} 2 & -1 \\ 4 & 3 \end{pmatrix}$.

(g) For what values of k the three vectors (1, 2, 2), (k, 1, 2) and (2, 2, 1) are linearly independent?

(h) Write the standard basis of $\mathbb{R}^2$ and $\mathbb{R}^3$.

(i) Prove that $S = \{(x, y, z) \in \mathbb{R}^3 / x + y + z = 0\}$ is a subspace of $\mathbb{R}^3$.

2. (a) If u and v are linearly independent vectors in a vector space V then show that so are $u+v$ and $u-v$. 2

(b) Examine whether the set of vectors are linearly independent in $\mathbb{R}^3$. 4

$$\{(1, 2, 3),\ (2, 3, 1),\ (3, 1, 2)\}.$$

(c) Define Dilation and Rotation. 2

3. (a) Let A be a singular matrix. Is 0 is an eigen value of A? Justify your answer. 4

   (b) If λ be an eigen value of a non-singular matrix A, then λ^{-1} is an eigen value of A^{-1}. 4

4. (a) Define Eigen space and invariant space with examples. 2+2

   (b) Show that the matrix $A=\begin{pmatrix}1 & 0\\ 3 & 1\end{pmatrix}$ is not diagonalisable. 4

5. (a) Define a basis of a vector space. Do the vectors (1, 0, 0), (0, 1, 0) and (1, 2, 1) are form a basis of $\mathbb{R}^3$? Justify. 4

   (b) Find the eigen values and corresponding eigen vectors of the following real matrix. 4

$$\begin{pmatrix}2 & 0 & 0\\ 0 & 3 & 0\\ 0 & 0 & 5\end{pmatrix}$$

6. (a) Prove or disprove: The set $\{(x, y, z) \in \mathbb{R}^3 \mid ax+by+cz=0 \text{ and } a^2+b^2+c^2 \neq 0\}$ is a subspace of $\mathbb{R}^3$. 3

   (b) Use elementary row operations on A to obtain A^{-1} where 5

$$A=\begin{bmatrix}2 & 0 & 0\\ 4 & 3 & 0\\ 6 & 4 & 1\end{bmatrix}$$

7. (a) Let $A=\begin{bmatrix}3 & 2 & -6\\ 0 & -1 & 4\\ 5 & -2 & 0\end{bmatrix}$. Verify that $A+A'$ is symmetric and $A-A'$ is skew-symmetric and hence express A as the sum of a symmetric and skew-symmetric matrix. 4

   (b) If $A=\begin{bmatrix}1 & 0 & 2\\ 0 & -1 & 1\\ 0 & 1 & 0\end{bmatrix}$, then verify that A satisfies its own characteristic equation. Hence find A^{-1} and A^9. 4

8. (a) Find a basis of $\mathbb{R}^3$ containing the vectors (1, 1, 0) and (1, 1, 1). 3

   (b) Let A, B, C be three square matrices such that $A \neq O$ and $AB = AC$, where O is the null matrix. Does it imply $B = C$? Justify your answer. 3

   (c) Define dimension of a finite dimensional vector space V over the field F. Give example. 2

9. (a) Find the 3 by 3 matrix representations of the following transformations. 2+2

(i) projection of any point on the x-y plane.

(ii) reflection of any point through the x-y plane.

(b) Determine the rank of $A = \begin{pmatrix} x & 1 & 0 \\ 3 & x-2 & 1 \\ 3(x+1) & 0 & x+1 \end{pmatrix}$, for different values of x. 4

10.(a) Solve by matrix method: 4

$$x + y + z = 4,$$
$$2x - y + 3z = 1,$$
$$3x + 2y - z = 1.$$

(b) Reduce the matrix to fully reduced normal form 4

$$\begin{pmatrix} 1 & 0 & 2 & 3 \\ 2 & 0 & 4 & 6 \\ 3 & 0 & 7 & 2 \end{pmatrix}.$$

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