WEST BENGAL STATE UNIVERSITY

B.Sc. Programme 6th Semester Examination, 2021

MTMGDSE04T-MATHEMATICS (DSE2)

LINEAR PROGRAMMING

Time Allotted: 2 Hours | Full Marks: 50

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1. Answer any ***five*** questions from the following: 2×5 = 10

(a) Find the extreme points of the following convex set $S=\{(x,\ y):\ y^2 \leq 4x\}$.

(b) Find a supporting hyperplane of the convex set

$$S=\{(x,\ y):\ x+2y\leq 4,\ \ 3x+y\leq 6,\ \ x\geq 0,\ \ y\geq 0\}$$

(c) In the following equations find the basic solution with x_3 as the non-basic variable.

$$x_1+4x_2-x_3=3$$
$$5x_1+2x_2+3x_3=4$$

(d) Is $(2\ ,\ 0)$ a feasible solution of the following LPP?

Maximize $\quad Z=x_1+3x_2$

Subject to $\quad 3x_1+6x_2\leq 8$

$5x_1+2x_2\leq 10$

$x_1,\ x_2\geq 0$

(e) Write the following problem in a standard form:

Maximize $\quad Z=x_1+x_2+x_3$

Subject to $\quad |x_1-x_2+x_3|\leq 2$

$x_1-x_2-x_3=3$

$x_1,\ x_2,\ x_3\geq 0$

(f) When a basic solution is said to be degenerated?

(g) A hyperplane is given by $x+3y+2z=9$. In which half spaces the points $(1,\ 2,\ 4)$ and $(-3\ ,1\ ,-5)$ lie?

(h) Give an example of a non-convex set. Explain why it is non-convex.

(i) Find the number of basic feasible solution of the following LPP:

Maximize $Z = 2x_1 + 3x_2$

Subject to $x_1 + x_2 \geq 2$

$x_1 - x_2 \leq 1$

$x_1,\ x_2 \geq 0$

2. (a) Find all basic feasible solution of the following system of equations. 4

$$x_1 + x_2 - x_3 = 2,$$
$$2x_1 + x_2 - x_3 = 3$$

(b) Show that $(1, 2, 1)$ is a feasible solution of the system of equations. 4

$$x_1 - x_2 + 2x_3 = 1$$
$$x_1 + 2x_2 - x_3 = 4$$
$$x_1,\ x_2,\ x_3 \geq 0$$

Reduce the feasible solution to a basic feasible solution.

3. (a) Solve: 4

Maximize $Z = 2x_1 - 3x_2$

Subject to $x_1 + x_2 \leq 2$

$2x_1 + 2x_2 \geq 8$

$x_1,\ x_2 \geq 0$

(b) Food X contains 6 units of vitamin A and 7 units of vitamin B per gram and costs 12 p./gm. Food Y contains 8 units of vitamin A and 12 units of vitamin B per gram and costs 20 p./gm. The daily requirements of vitamin A and B are at least 100 units and 120 units respectively. Formulate the above as an L.P.P. to minimize the cost. 4

4. (a) If the feasible region of a linear programming problem is strictly bounded and contains a finite number of extreme points then prove that the objective function of the linear programming problem assumes its optimal value at an extreme point of the convex set of feasible solutions. 4

(b) Show that the feasible solution $x_1 = 1,\ x_2 = 1,\ x_3 = 0,\ x_4 = 2$ to the system 4

$$x_1 + x_2 + x_3 = 2$$
$$x_1 + x_2 - 3x_3 = 2$$
$$2x_1 + 4x_2 + 3x_3 - x_4 = 4$$
$$x_1,\ x_2,\ x_3,\ x_4 \geq 0$$

is not basic.

5. (a) Prove that the set of all convex combinations of a finite number of points is a convex set. 5

(b) Give an example of

(i) Convex hulls in E^2 and E^3

(ii) Convex polyhedron in E^2

(iii) Simplex in zero and one dimension.

6. (a) Prove that in a linear programming problem the optimal hyperplane is a supporting hyperplane to the convex set of feasible solution. 4

(b) Find a supporting hyperplane passing through $(7, -1)$ of the convex set $X = \{(x_1, x_2) : x_1^2 + x_2^2 \le 25\}$. 4

7. Solve the following L.P.P. using duality theory. 8

Maximize $Z = 4x_1 + 3x_2$

Subject to $x_1 \le 6$

$x_2 \le 8$

$x_1 + x_2 \le 7$

$3x_1 + x_2 \le 15$

$-x_2 \le 1$

$x_1, x_2 \ge 0$

8. (a) Solve by Charnes Big M-method the following L.P.P. 6

Minimize $Z = 4x_1 + 2x_2$

Subject to $3x_1 + x_2 \ge 27$

$x_1 + x_2 \ge 21$

$x_1 + 2x_2 \ge 30$

$x_1, x_2 \ge 0$

(b) Discuss whether the set of points $(0, 0)$, $(0, 1)$, $(1, 0)$, $(1, 1)$ on the xy-plane is a convex set or not. 2

9. (a) Given the L.P.P. 5

Maximize $Z = 2x_1 + 3x_2 + 4x_3$

Subject to $x_1 - 5x_2 + 3x_3 = 7$

$2x_1 - 5x_2 \le 3$

$3x_2 - x_3 \ge 5$

$x_1, x_2 \ge 0$

x_3 is unrestricted in sign. Formulate the dual of the L.P.P.

(b) Prove that if any variable of the primal problem be unrestricted in sign, then the corresponding constraint of the dual will be equality. 3

10.(a) If for a basic feasible solution x_B of a linear programming problem maximi $z = cx$, subject to $Ax = b$ and $x \geq 0$, we have $z_j - c_j \geq 0$ for every column a_j A, then prove that x_B is an optimal solution.

(b) Check whether $x = 5,\ y = 0,\ z = -1$ is a basic solution of the system of equations. 3

$$x + 2y + z = 4,$$
$$2x + y + 5z = 5$$

N.B. : *Students have to complete submission of their Answer Scripts through E-mail / Whatsapp to their own respective colleges on the same day / date of examination within 1 hour after end of exam. University / College authorities will not be held responsible for wrong submission (at in proper address). Students are strongly advised not to submit multiple copies of the same answer script.*

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